Initial Deceleration and Late-Time Acceleration of the Universe

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In this paper we present a higher dimensional (5D) spatially-flat homogeneous cosmological model in presence of time dependent Λ.This model with a particular form of Hubble's parameter represents an early deceleration and late-time acceleration. It has been found that with the evolution of the universe the ordinary space dimensions expand whereas the extra dimension gets compactified with recent observation (Chodos and Detweller). The physical behaviors of the model are discussed.

1. Introduction

There are evidences in favour of the fact that the universe at present is expanding with acceleration. The recent observations of large scale structure, such as the Hubble diagram of type 1a supernovae (SNe) and the angular power spectrum of the cosmic microwave background (CMB) indicate that the universe is flat and is in a stage of accelerated expansion [1,2]. The very recent WMAP data [3] also confirms this. The universe is dominated by a form of matter with negative pressure, which is widely referred to as dark energy today and it leads to cosmic acceleration. The mechanism causing this accelerated expansion is still not known. The dark energy occupies about 73% of the energy of our universe, while dark matter occupies about 23% and the usual baryonic matter 4%. One of the candidates for this dark energy is the cosmological constant or a time varying cosmological parameter $\Lambda(t)$ [4]. A kind of repulsive force which acts as anti-gravity is responsible for accelerating universe. At present, Λ with a dynamic character is preferred over a constant Λ, specially a time dependent Λ, which decreases slowly from its large value to reach its small value at present era [5]. An intensified research is going on to find the true nature of this acceleration. A scalar field ϕ with a potential $V(\phi)$, which is known as quintessence [6] decreases slowly with time, may be another candidate for dark energy. Quintessence exerts negative pressure and is dynamic in nature. There are other candidates of dark energy viz. K-essence [7], tachyon [8], phantom [9, 10] etc. Attempts are also being made by a section of workers to recast the theory of relativity in a higher dimensional space time. Also, modern developments of superstring theory and Yang-Mills supergravity in its field theory need higher dimensional space time. In years there has been considerable interest in theories with higher dimensional space time, in which extra dimensions are contracted to a very small size, apparently beyond our ability for measurement. A model of higher dimension was proposed by Kaluza and Klein [11, 12] who tried to unify gravity with electromagnetic interaction by introducing an extra dimension, which is an extension of Einstein General Relativity in five dimensions. The activities in extra dimensions also stem from the Space-Time-Matter (STM) theory proposed by Wesson et al. [13]. In recent years a number of authors [14-17] have considered multi dimensional cosmological models. A higher-dimensional FRW type model [18] is considered where the acceleration is caused apparently by the presence of extra dimension.

In this paper, we have proposed a form of Hubble's parameter H as a function of the average scale factor S that leaves us with a model of the universe which starts with a decelerating expansion and changes from decelerating phase to accelerating one at late times. Singh [19] derived a four-dimensional cosmological model. Our model is a five-dimensional one. This type of investigations had earlier been used by Ellis and Madsen [20] for finding out the potential driving inflation i.e., an accelerated phase of the universe at a very early stage of its evolution. Our paper is organized as follows. After Introduction, in Sec.1, we present the field equations for a higher-dimensional (i.e., 5D) spatially-flat homogeneous cosmological model in Sec. 2. In Sec. 3 of this paper we obtain the solutions of the field equations.

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In Sec. 4 we discuss the physical nature of the model. The paper ends with concluding remarks in Sec. 5.

2. Field Equations

We discuss here a spatially flat five dimensional homogeneous cosmological model described by the line element

$$ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] - b^2(t)\,dy^2 \quad (1)$$

Where, $a(t)$ is the scale factor for the three dimensional space and $b(t)$ that for the extra dimension and y is the fifth dimensional co-ordinate.

The Einstein field equations are given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} + \Lambda g_{ij} \quad (2)$$

Where, $8\pi G = c = 1$ and R_{ij} are the Ricci tensors and T_{ij} are the energy momentum tensors, which may be written as follows [21]

$$T_{00} = \rho(t)\ \ T_{11} = T_{22} = T_{33} = p(t),\ \ T_{44} = p_H\, g_{44} \quad (3)$$

Where, p_H is the pressure in the fifth dimension and ρ and p are the density and pressure for isotropic three dimensional space.

The Einstein field equations for the metric in Eqn. (1) are

$$3\frac{\dot{a}^2}{a^2} + 3\frac{\dot{a}\dot{b}}{ab} = \rho + \Lambda \quad (4)$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{b}}{b} = -p + \Lambda \quad (5)$$

$$3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} = -p_H + \Lambda \quad (6)$$

Covariant divergence of Eqn. (2) gives

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) + (\rho + p_H)\frac{\dot{b}}{b} + \dot{\Lambda} = 0 \quad (7)$$

We assume that $p = p_H$ i.e., the pressure is isotropic including the extra space.

The equation of state is

$$p = p_H = \omega\rho \quad (8)$$

From Eqns. (5) and (6), we get

$$\frac{\dot{a}}{a} - \frac{\dot{b}}{b} = \frac{k_1}{S^4} \quad (9)$$

We define the average scale factor S as

$$S^4 = a^3 b \quad (10)$$

We introduced the volume expansion θ and shear scalar σ as usual

$$\theta = v^i_{;i} \quad ; \quad \sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \quad (11)$$

σ^{ij} being shear scalar.

In the above equations, the semicolon stands for covariant differentiation. For this 5D metric, expression for dynamical scalars comes to be

$$\theta = 4\frac{\dot{S}}{S} \quad (12)$$

$$\sigma = \frac{k_2}{S^4} \quad (13)$$

Where, $k_2 (> 0)$ is a constant.

The Hubble parameter H and the deceleration parameter q are defined as

$$H = \frac{\dot{S}}{S} \quad (14)$$

$$q = -\frac{\ddot{S}}{SH^2} \quad (15)$$

Eqns. (4), (5), (6) and (7) can be written in terms of H, σ and q as

$$\rho + \Lambda = 6H^2 - \sigma^2 \quad (16)$$

$$p - \Lambda = 3(q-1)H^2 - \sigma^2 \quad (17)$$

and

$$\dot{\rho} + \frac{\dot{S}}{S}(\rho + p) + \dot{\Lambda} = 0 \quad (18)$$

In order for an expanding model of the universe to be consistent with the observation, one needs a Hubble parameter H such that the model starts with a decelerating expansion followed by an accelerating expansion at late times. Following the lines floated by Ellis and Madsen (1991), we choose a variation form for H, which describe both decelerating and an accelerating phase of the universe, which yields

$$H(S) = m(S^{-n} + 1) \tag{19}$$

Where, $m > 0$ and $n (> 1)$ are constants.

For this choice the deceleration parameter q becomes

$$q = \frac{n}{(S^n + 1)} - 1 \tag{20}$$

From Eqn. (20) we observe that when $S = 0, q = n - 1 > 0$ and $q = 0$ implies $S^n = n - 1$. Therefore, for $S^n > n - 1, q < 0$.

The Universe begins with a decelerating expansion and the expansion changes from decelerating phase to accelerating one.

Using Eqns. (11), (19) and (20) in Eqns. (16) and (17)

$$\rho = 6m^2 (S^{-n} + 1)^2 - \frac{k_2}{S^8} - \Lambda \tag{21}$$

$$p = m^2 (S^{-n} + 1)^2 \left\{ \frac{3n}{(S^n + 1)} - 6 \right\} - \frac{k_2}{S^8} + \Lambda \tag{22}$$

It is observed that the model has singularity at t = 0, (i.e., S = 0).

From Eqns. (8), (16) and (17)

$$\frac{\Lambda}{H^2} = \frac{3}{2}(3 - q) + \frac{(\omega - 1)}{2} \frac{\rho}{H^2} \tag{23}$$

Hence $\Lambda > 0$ for $q < 3 + \frac{(\omega-1)}{3}\frac{\rho}{H^2}$ and $\Lambda < 0$ for $q > 3 + \frac{(\omega-1)}{3}\frac{\rho}{H^2}$.

From Eqns. (19) and (20)

$$\frac{1}{H} = \frac{n - 1 - q}{mn} \tag{24}$$

H^{-1} increases as q decreases. The maximum of H^{-1} is m^{-1} for $q = -1$. Also from Eqn. (13) we obtained $\dot{\sigma} = -4\sigma H$, thus the energy density associated with the anisotropy σ decays rapidly in an evolving universe and it becomes negligible for infinitely large value of average scale factor S.

3. Solutions of the Field Equations

The field equations are solved by using the assumption in Eqn. (19). We take Eqn. (19) as our key equation for solving the field equations.

Eqn. (19) on integration yields

$$S^n = e^{nmt + k_0} - 1 \tag{25}$$

Where, k_0 is the constant of integration.

Setting $S = 0$ for $t = 0$ we obtain, $k_0 = 0$.

Hence

$$S^n = e^{nmt} - 1 \tag{26}$$

Eqns. (8), (19), (21) and (22) yield

$$\rho = \frac{1}{(1+\omega)} \left[\frac{3n\, m^2 e^{nmt}}{(e^{nmt} - 1)^2} - \frac{2k_2}{(e^{nmt} - 1)^{\frac{8}{n}}} \right] \tag{27}$$

$$p = \frac{\omega}{(1+\omega)} \left[\frac{3n\, m^2 e^{nmt}}{(e^{nmt} - 1)^2} - \frac{2k_2}{(e^{nmt} - 1)^{\frac{8}{n}}} \right] \tag{28}$$

$$\Lambda = \frac{6m^2 e^{2mnt}}{(e^{mnt} - 1)^2} - \frac{3nm^2 e^{mnt}}{(1+\omega)(e^{mnt} - 1)^2} + \frac{k_2 (1 - \omega)}{(1+\omega)(e^{mnt} - 1)^{\frac{8}{n}}} \tag{29}$$

Expansion scalar θ, shear σ and deceleration parameter q for this higher-dimensional cosmological model become

$$\theta = \frac{4m}{(1 - e^{-mnt})} \tag{30}$$

$$\sigma = \frac{k_2}{(e^{mnt} - 1)^{\frac{4}{n}}} \tag{31}$$

and

$$q = \frac{n}{e^{mnt}} - 1 \tag{32}$$

Eqns. (9), (12) and (25) give the metric coefficients as

$$a(t) = k_3 (e^{mnt} - 1)^{1/n} \exp\left(\frac{k_1}{4} \int \frac{dt}{(e^{mnt} - 1)^{\frac{4}{n}}} \right) \tag{33}$$

and\

$$b(t) = k_4 (e^{mnt} - 1)^{1/n} \exp\left(-\frac{3k_1}{4} \int \frac{dt}{(e^{mnt} - 1)^{\frac{4}{n}}} \right) \tag{34}$$

Where, k_3 and k_4 are positive constants.

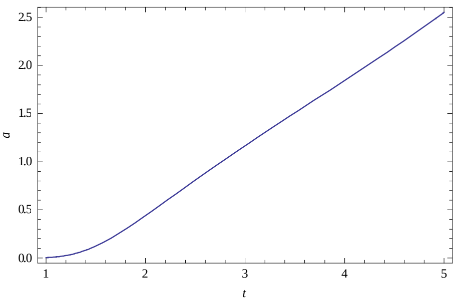

Fig.1.

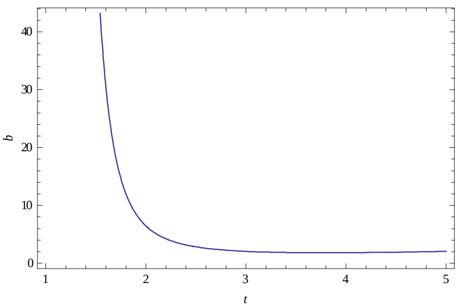

Fig.2.

Figs. 1 and 2 show variation of scale factors $a(t)$ and $b(t)$ versus cosmic time t by taking m = 0.17, $k_1 = 2$, $k_3 = k_4 = 1$ and n = 1.5.

Eqns. (33) and (34) indicate that the scale factor $a(t)$ is increasing while the higher-dimensional scale factor $b(t)$ is decreasing with the increase of cosmic time.

4. Discussions

This model has singularity at $t = 0$ and begins evolving with a big bang at $t = 0$ with large ρ, Λ, θ and σ and $q = n - 1 (> 0)$. For large time i.e., $t \to \infty, \rho$ and σ tend to zero and q, the deceleration parameter tends to -1. For $-1 < \omega \triangleleft -1/3$ [22], Eqns. (27) and (28) show ρ as positive and p as negative.

From Eqn. (32), equating $q = 0$ we can find the time t_q when the expansion changes from decelerating phase to accelerating phase.

$$t_q = \frac{\log n}{mn} \tag{35}$$

From Eqns. (19) and (26) we obtain the age of the universe t_0 as

$$t_0 = \frac{1}{mn} \log\left(\frac{H_0}{H_0 - m} \right) \tag{36}$$

Where, H_0 denotes the present day Hubble parameter.

The Universe starts expanding with a big bang singularity.

5. Concluding Remarks

In this paper, we have discussed five-dimensional homogeneous cosmological model. We know that the best candidates for unification of the forces of nature in a quantum gravitational environment seem to exist in finite form if there are many more dimensions of space than three that we are familiar with. We have taken only one extra spatial dimension, but we believe that most of the findings may be extended if we take a larger number of extra dimensions. The important findings may be discussed as we have used the proposal of a variation law for Hubble's parameter H as a function of average scale factor S by Ellis and Madsen in 5D FRW type space-time. The deceleration parameter, given by $q = n - 1$, provides accelerating universe for $n < 1$ and decelerating one for $n > 1$. Exact solutions of Einstein field equations have been obtained by using the variation law for H. This model evolves with decelerating expansion in the initial epoch followed by a late time accelerated expansion. Interestingly we observe that for $-1 \triangleleft \omega \triangleleft -1/3$ [22], the pressure becomes negative while the density is positive. One immediate conclusion is that the universe is dominated by a form of matter with negative pressure, which is widely referred to as dark energy today. From Eqns. (33) and (34) we have seen (Figs. 1 and 2) for this five dimensional cosmological model three spatial dimensions increased whereas the extra dimension is compact [23] as cosmic time proceed. The discussion of the model reveals that the universe may be dominated by dark energy, which can describe the accelerating nature of the universe.

Acknowledgments

The authors acknowledge the financial support of UGC, New Delhi and the Department of Mathematics, Gauhati University for all facilities for doing this work.

References

[1] A. G. Riess et al., Astron. J. **116**, 1009 (1998).
[2] S. Perlmutter et al., Astrophys J. **517**, 565 (1999).
[3] A. Balbi et al., Astrophys J. **545**, L 1 (2000); S. Hanany et al., *ibid*, **545**, L 5 (2000).
[4] E. Hawkins et al., Mon. Not. R. Astron. Soc. **346**, 78 (2003).
[5] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
[6] L. Verde et al., Mon. Not. R. Astron. Soc. **335**, 432 (2002) .
[7] R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998).
[8] A. R. Liddle and R. J. Scherrer, Phys. Rev. **D59**, 023509 (1999).
[9] P. J. Steinhard, L. M. Wang and T. Zlatev, Phys. Rev. **D59**, 123504 (1999).
[10] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. **B484**, 112 (2000).
[11] T. Kaluza, Prens. Acad. Wiss. **F1**, 9669 (1921).
[12] O. Klein, Z. Phys. **37**, 895 (1926).
[13] P. S. Wesson, *Space-Time-Matter* (World Scientific, Singapore, 1999).
[14] S. Chatterjee, D. Panigrahi and A. Banerjee, Class. Quant. Grav. **11**, 371 (1994).
[15] S. Chatterjee, and A. Banerjee, Class. Quant. Grav. **10**, L1 (1993).
[16] A. Banerjee, D. Panigrahi and S. Chatterjee, J. Math Phys. **36**, 3619 (1995).
[17] S. Chakraborty and A. Ghosh, Int. J. Mod. Phys. **D9**, 543 (2000).
[18] S. Chatterjee and A. Banerjee, arxiv, gr-qc/0509112v1 28sep 2005.
[19] J. P. Singh, Astrophys. Space Sci. **318**, 103 (2008).
[20] G. F. R. Ellis and M. Madsen, Class. Quant. Grav. **8**, 667 (1991).
[21] S. Randjibar Daemi, A. Salam and J. Strathdee, Phys. Lett. **B135**, 388 (1984).
[22] S. D. Katore, Prespacetime Journal **2**, 1232 (2011).
[23] A. Chodos and S. Detweller, Phys. Rev. **D21**, 2167 (1980).

Received: 1 January, 2013
Accepted: 10 May, 2013