# Bianchi Type I Holographic Polytropic Gas Model of Dark Energy with Hybrid Expansion Law

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In this paper, we consider Bianchi type I universe filled with non-interacting cold dark matter and holographic dark energy. Exact solutions of Einstein's field equations are obtained by assuming the average scale factor to be a combination of power law and exponential function, known as hybrid expansion law. The physical and geometrical properties of the solutions are also discussed. The results obtained are found to be consistent with recent cosmological observations. Moreover, a correspondence between the holographic dark energy and polytropic gas model of dark energy is established which allows us to reconstruct the potential and the dynamics for the scalar field of the polytropic gas describing the accelerated expansion of the Universe.

## 1. Introduction

According to various recent cosmological and astrophysical observations such as Supernovae Type Ia (SN Ia) [1, 2, 3], Cosmic Microwave Background (CMB) [4, 5], Large Scale Structure (LSS) [6, 7, 8] and other, it is evident that our universe is currently undergoing an accelerated phase of expansion. In the framework of standard cosmology, an exotic component with negative pressure, dubbed dark energy (DE), is needed to explain this acceleration.

Many candidates of dark energy are proposed in the literature. Among them the cosmological constant with the equation of state parameter $\omega_{\Lambda} = -1$ is the earliest, simplest and the most natural candidate of dark energy, and the ΛCDM (Λ-cold dark matter) is the most successful model for the present accelerated expanding universe. However, from theoretical viewpoint it faces with the fine-tuning and cosmic coincidence problems [9].

In addition to the cosmological constant, there are dynamical dark energy scenarios to explain the nature of dark energy such as quintessence [10 - 12], phantom [13], k-essence [14], tachyon [15], dilatonic ghost condensate model [16]. Some interacting models of dark energy such as Chaplygin gas models [17], brane-world models [18] etc. are also considered in the literature.

There has been also considerable interest to explain the observed acceleration of the Universe with the help of quantum gravitational principle i.e., the holographic dark energy principle. The principle was first put forwarded by G. ’t Hooft [19] to explain the thermodynamics of black hole physics. Holographic dark energy model emerges from the Holographic Principle which states that the number of degrees of freedom directly related to entropy of the system scales with the enclosing surface area of the system and not with its volume [20]. A new version of this holographic principle was first applied by Fischler and Susskind [21] to cosmological context which states that the gravitational entropy within a closed surface should not be always larger than the particle entropy that passes through the past light-cone of that surface.

But the IR cut-off cannot be determined with the help of holographic dark energy principle. The researchers working in this field proposed various choices of IR cut-off which lead to new problems in physics. Granda and Oliveros [22] proposed a holographic dark energy density of the form $\rho_{HDE} \approx \alpha H^2 + \beta \dot{H}$, where $H$ is the Hubble parameter, $\alpha$ and $\beta$ are constants which must satisfy the restrictions imposed by the current observational data. They showed that this new dark energy model can explain the current cosmic acceleration of our universe and is consistent with the observational data. Granda and Oliveros [23] have established correspondence between quintessence, tachyon, k-essence and dilaton dark energy models with this holographic dark energy in the flat Friedman Robertson Walker (FRW) universe. Chattopadhyay [24], Farajollahi *et al.* [25], Karami and Fehri [26], Malekjani [27], Rao *et al.* [28], Guberina *et al.* [29], Iv`an and Pav`on [30], Mete *et al.* [31], Ghaffari [32], Rahman and Ansari [33], Saridakis [34], Saadat [35], Srivastava *et al.* [36], Katore and Kapse [37] *et al.* have also investigated several aspects of holographic dark energy (HDE).

It is a well-known fact that although our universe is homogeneous and isotropic at large scale, and one of the most generalizations of the flat universe is the FRW model of the Universe, but there is no observational evidence that rules out the possibility of an anisotropic universe. The issue of global anisotropy can be settled if we can incorporate anisotropy to the flat FRW model in a suitable manner [38]. The FRW universe has the same scale factor for each of the three spatial directions. The Bianchi type I universe has different scale factors in the three spatial directions. Also, it is the simplest spatially homogeneous and anisotropic flat universe. Spatially homogeneous and anisotropic cosmological models play a significant role in the description of large scale behavior of the universe and such models have been widely studied by many authors in search of a relativistic picture of the early universe.

In literature, various Bianchi type models are studied in different contexts. Bianchi type I homogeneous models are the simplest anisotropic models of the universe whose spatial sections are flat, but the expansion or contraction rates are directionally dependent. For a simplification and description of the large scale structure and behavior of the actual universe, anisotropic Bianchi type I models have been considered by several authors on different aspects. So it will be interesting to study the evolution of the universe with non interacting cold dark matter and holographic dark energy in an anisotropic universe like Bianchi type I.

In stellar astrophysics, the polytropic gas model can explain the equation of state of degenerate white dwarfs, neutron stars and also the main sequence stars [39]. They have constructed the equation of state of polytropic gas as $P_{pg} = K\rho_{pg}^{1+\frac{1}{\eta}}$, where *K* and η are the polytropic constant and polytropic index, respectively. The idea of dark energy with polytropic gas equation of state has been investigated by U. Mukhopadhyay and S. Ray [40] in cosmology. Karami *et al.* [41] have constructed energy density and pressure corresponding to scalar field $\phi$ for polytropic gas. Rahman and Ansari [33], Karami *et al*. [41], Karami and Ghaffari [42], Adhav [43], Setare and Kamali [44], Taji and Malekjani [45] and many other authors have investigated polytropic gas in different models to explain the late time cosmic acceleration.

In this paper, we consider the spatially homogeneous and anisotropic Bianchi type I universe filled with non-interacting cold dark matter and holographic dark energy and investigate the correspondence with polytropic gas model. The paper is organized as follows: In Sec. 2, we derive the cosmic evolution equations from the Einstein field equations in the background of Bianchi type I metric. Cosmological solutions of the field equations are obtained in Sec. 3 by taking average scale factor to be a combination of power law and exponential function which is termed as hybrid expansion law. We present our solutions in Sec. 4 with a brief discussion. The correspondence between holographic dark energy and polytropic gas model is established in Sec. 5. We conclude the paper with a brief discussion in Sec. 6.

## 2. The metric and field equations

We consider the spatially homogeneous and anisotropic Bianchi type I space-time described by the line element

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + +C^2dz^2 \qquad (1)$$

Where, *A*, *B*, *C* are functions of cosmic time *t* only.

We assume that the universe is filled with non-interacting cold dark matter and holographic dark energy (HDE).

Einstein's field equations in natural units ($8\pi G = 1, c = 1$) are given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -(T_{ij} + \bar{T}_{ij}) \qquad (2)$$

Where, $R_{ij}$ is the Ricci tensor, $R$ is the Ricci scalar, $T_{ij}$ and $\bar{T}_{ij}$ are the energy momentum tensors for cold dark matter and HDE respectively.

The energy momentum tensor $T_{ij}$ for cold dark matter with energy density $\rho_m$ is given by

$$T_{ij} = \rho_m u_i u_j \qquad (3)$$

And the energy momentum tensor $\bar{T}_{ij}$ for HDE is given by

$$\bar{T}_{ij} = (\rho_{HDE} + p_{HDE})u_i u_j + g_{ij}p_{HDE} \qquad (4)$$

Where, $\rho_{HDE}$ and $p_{HDE}$ are the energy density and the pressure of the HDE, respectively.

The HDE density proposed by L. N. Granda and A. Oliveros [18] is

$$\rho_{HDE} = 3M_p^2(\alpha H^2 + \beta \dot{H}) \qquad (5)$$

Where, $M_p^{-2} = 8\pi G = 1$ and α and β are constants.

Now, in co-moving coordinate system the equations (2) with (3) and (4) for the metric (Eqn. (1)) lead to the following system of field equations

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -p_{HDE} \qquad (6)$$

$$\frac{\ddot{C}}{C}+\frac{\ddot{A}}{A}+\frac{\dot{C}\dot{A}}{CA}=-p_{HDE} \quad (7)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=-p_{HDE} \quad (8)$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}=\rho_m+\rho_{HDE} \quad (9)$$

Where, an over dot denotes differentiation with respect to the cosmic time *t*.

The conservation of energy-momentum yields

$$\dot{\rho}_m+\dot{\rho}_{HDE}+3H(\rho_m+\rho_{HDE}+p_{HDE})=0 \quad (10)$$

But the continuity equation for the cold dark matter is

$$\dot{\rho}_m+3H\rho_m=0 \quad (11)$$

And the continuity equation of the HDE is

$$\dot{\rho}_{HDE}+3H(\rho_{HDE}+p_{HDE})=0 \quad (12)$$

The equation of state parameter for HDE is

$$\omega_{HDE}=\frac{p_{HDE}}{\rho_{HDE}} \quad (13)$$

Therefore, from Eqns. (5), (12) and (13), we get

$$\omega_{HDE}=-1-\frac{2\alpha H\dot{H}+\beta\ddot{H}}{3H(\alpha H^2+\beta\dot{H})} \quad (14)$$

## 3. Cosmological solutions of the field equations

From equations (6) - (9), we derive

$$A(t)=a_1(ABC)^{\frac{1}{3}}\exp(b_1\int(ABC)^{-1}dt) \quad (15)$$

$$B(t)=a_2(ABC)^{\frac{1}{3}}\exp(b_2\int(ABC)^{-1}dt) \quad (16)$$

$$C(t)=a_3(ABC)^{\frac{1}{3}}\exp(b_3\int(ABC)^{-1}dt) \quad (17)$$

Where,

$a_1a_2a_3=1$ and $b_1+b_2+b_3=0$

To find an exact solution, we consider the average scale factor $a$ given by

$$a=(ABC)^{\frac{1}{3}} \quad (18)$$

as

$$a(t)=a_0(\frac{t}{t_0})^{\gamma}e^{\xi(\frac{t}{t_0}-1)} \quad (19)$$

Where, $\gamma$ and $\xi$ are non-negative constants and $a_0$ and $t_0$ represent the present value of the scale factor and age of the Universe, respectively. The constants $\gamma=0$ yields the exponential law cosmology and $\xi=0$ gives power law cosmology. Thus, the relation in Eqn. (19) is a combination of a power-law and an exponential function and therefore, it is called the Hybrid Expansion Law (HEL). This law was first proposed by Akarsu *et al.* [46]. Further, the scale factor given by Eqn. (18) yields a time-dependent deceleration parameter which exhibits a transition of the Universe from the early decelerating phase to the present accelerating phase.

Now, using Eqns. (19) in (15), (16) and (17) we get

$$A(t)=a_1(kt^{3\gamma}e^{\frac{3\xi t}{t_0}})^{\frac{1}{3}}\exp(b_1F(t)) \quad (20)$$

$$B(t)=a_2(kt^{3\gamma}e^{\frac{3\xi t}{t_0}})^{\frac{1}{3}}\exp(b_2F(t)) \quad (21)$$

$$C(t)=a_3(kt^{3\gamma}e^{\frac{3\xi t}{t_0}})^{\frac{1}{3}}\exp(b_3F(t)) \quad (22)$$

Where,

$F(t)=\int(kt^{3\gamma}e^{\frac{3\xi t}{t_0}})^{-1}dt$ and k is a non zero constant of integration.

Hence, the line element (1) can be expressed as

$$ds^2=-dt^2+(kt^{3\gamma}e^{\frac{3\xi t}{t_0}})^{-\frac{2}{3}}[a_1^2\exp(2b_1F(t))\,dx^2+a_2^2\exp(2b_2F(t))\,dy^2+a_3^2\exp(2b_3F(t))\,dz^2 \quad (23)$$

## 4. Result and discussion

The cosmological parameters viz. the spatial volume $(V)$ the directional Hubble parameters $(H_i)$, mean Hubble parameter $(H)$, expansion scalar ($\theta$), deceleration parameter ($q$), shear scalar ($\sigma^2$) and anisotropy parameter $(A)$ for our model are obtained as

$$V=a^3=[a_0\left(\frac{t}{t_0}\right)^{\gamma}e^{\xi\left(\frac{t}{t_0}-1\right)}]^3 \quad (24)$$

$$H_1=\frac{\dot{A}}{A}=\frac{1}{3}(kt^{3\gamma}e^{\frac{3\xi t}{t_0}})^{-1}k[3\gamma t^{3\gamma-1}e^{\frac{3\xi t}{t_0}}+t^{3\gamma}e^{\frac{3\xi t}{t_0}}\frac{3\xi}{t_0}]+b_1F'(t) \quad (25)$$

$$H_2=\frac{\dot{B}}{B}=\frac{1}{3}(kt^{3\gamma}e^{\frac{3\xi t}{t_0}})^{-1}k[3\gamma t^{3\gamma-1}e^{\frac{3\xi t}{t_0}}+t^{3\gamma}e^{\frac{3\xi t}{t_0}}\frac{3\xi}{t_0}]+b_2F'(t) \quad (26)$$

$$H_3=\frac{\dot{C}}{C}=\frac{1}{3}(kt^{3\gamma}e^{\frac{3\xi t}{t_0}})^{-1}k[3\gamma t^{3\gamma-1}e^{\frac{3\xi t}{t_0}}+t^{3\gamma}e^{\frac{3\xi t}{t_0}}\frac{3\xi}{t_0}]+b_3F'(t) \quad (27)$$

$$H=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=\frac{\gamma}{t}+\frac{\xi}{t_0} \quad (28)$$

$$\theta=3H=3(\frac{\gamma}{t}+\frac{\xi}{t_0}) \quad (29)$$

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=-1-\frac{\dot{H}}{H^2}=-1+\frac{\gamma}{(\gamma+\frac{\xi t}{t_0})^2} \quad (30)$$

$$\sigma^2=\frac{1}{2}(\sum_{i=1}^{3}H_i^2-\frac{1}{3}\theta^2)=\frac{M}{2(kt^{3\gamma}e^{\frac{3\xi t}{t_0}})} \quad (31)$$

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{M}{3}\frac{\left(kt^{3\gamma}e^{\frac{3\xi t}{t_0}}\right)^{-2}}{\left(\frac{\gamma}{t}+\frac{\xi}{t_0}\right)^2} \quad (32)$$

Where, $M = b_1^2 + b_2^2 + b_3^2$

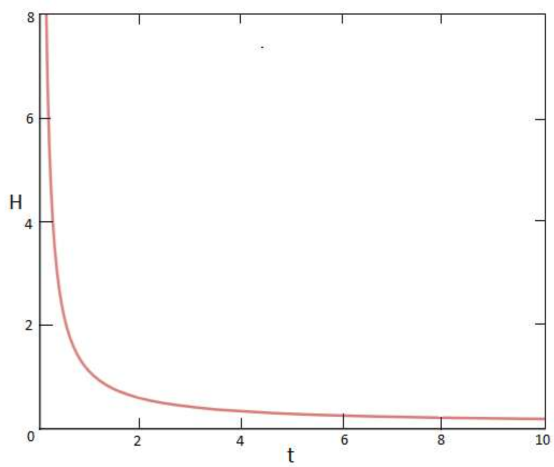


Fig. 1 $H$ vs. cosmic time t graph with $\gamma = 1, \xi = 1$ , $t_0 = 13.8$

Fig. 1 clearly shows that Hubble parameter is a decreasing function of cosmic time.

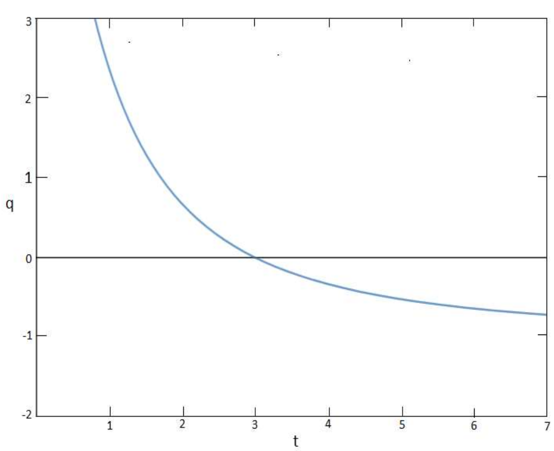


Fig. 2 Plot of deceleration parameter q vs. t graph with $\gamma = .1, \xi = 1, t_0 = 13.8$

Fig. 2 shows the variation of deceleration parameter ($q$) vs. cosmic time $t$. From the graph we see that the deceleration parameter (DP) decreases rapidly and approaches −1 asymptotically which shows de-Sitter like expansion at late time. For this model, the DP gives a transition from a decelerating expansion phase to the present accelerating phase of the universe. Also it is clear that the value of DP is positive at the early stage of the universe and becomes negative at late time. The negative value of DP shows the accelerating expansion of the universe. The Planck collaboration results (Ade et al. 2013) [47] shows that the value of the deceleration parameter lies in the range $-1 < q < 0$. Thus our derived model is suitable to describe the late time evolution of the universe.

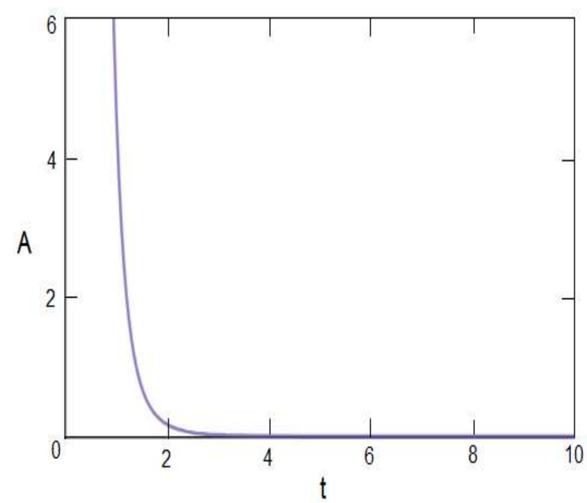


Fig. 3 Plot of anisotropy parameter vs. cosmic time t graph with $M = 1, k = .2, \gamma = 1, \xi = 1, t_0 = 13.8$

Fig. 3 shows the variation of anisotropic parameter ($A$) vs. cosmic time $t$. From the figure we see that anisotropic parameter decreases as time evolves and tends to zero. Hence, we can conclude that although our model is anisotropic at the early phase of the universe, the anisotropy dies out with time leading to the present isotropic phase of the Universe.

Now, using (28) in (5), we get

$$\rho_{HDE} = 3\left[\alpha\left(\frac{\gamma}{t}+\frac{\xi}{t_0}\right)^2 - \frac{\beta\gamma}{t^2}\right] \quad (33)$$

Again using (28) in (11), we get

$$\rho_m = D\left[t^{-3\gamma}e^{\frac{-\xi t}{t_0}}\right] \quad (34)$$

Where, $D$ is a constant of integration.
Thus the coincidence parameter is obtained as

$$\mu = \frac{\rho_{HDE}}{\rho_m} = \frac{3\left[\alpha\left(\frac{\gamma}{t}+\frac{\xi}{t_0}\right)^2 - \frac{\beta\gamma}{t^2}\right]}{D\left[t^{-3\gamma}e^{\frac{-\xi t}{t_0}}\right]} \quad (35)$$

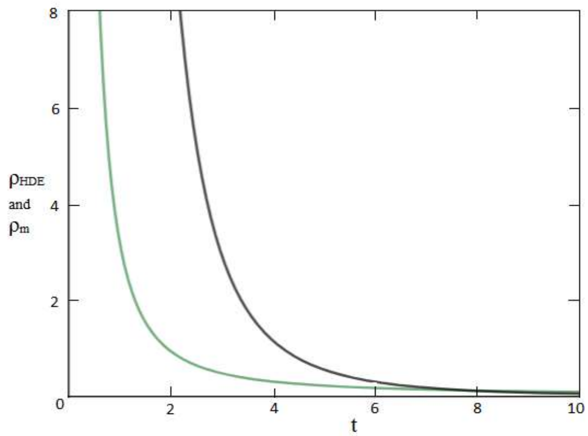


Fig 4 Holographic dark energy (HDE) density and dark matter density vs. cosmic time t graph with $\alpha=1, \gamma=1, \xi=1, \beta=0.05, D=100, t_0=13.8$ . Green line represents the HDE density and black line represents the cold dark matter density.

Fig. 4 shows that both HDE density and cold dark matter density decreases as time evolves while $\rho_m$ tends to zero and $\rho_{HDE}$ is near to zero at late time.

The dark matter density parameter ($\Omega_m$) and HDE density parameter($\Omega_{HDE}$) are given by

$$\Omega_m = \frac{D[t^{-3\gamma} e^{\frac{-\xi}{t_0}}]}{3\left(\frac{\gamma}{t}+\frac{\xi}{t_0}\right)^2} \tag{36}$$

$$\Omega_{HDE} = \alpha - \frac{\beta\gamma}{t^2\left(\frac{\gamma}{t}+\frac{\xi}{t_0}\right)^2} \tag{37}$$

Hence the total energy density parameter is given by

$$\Omega = \Omega_m + \Omega_{HDE} = \alpha + \frac{Dt^2\left[t^{-3\gamma} e^{\frac{-\xi t}{t_0}}\right] - 3\beta\gamma}{3t^2\left(\frac{\gamma}{t}+\frac{\xi}{t_0}\right)^2} \tag{38}$$

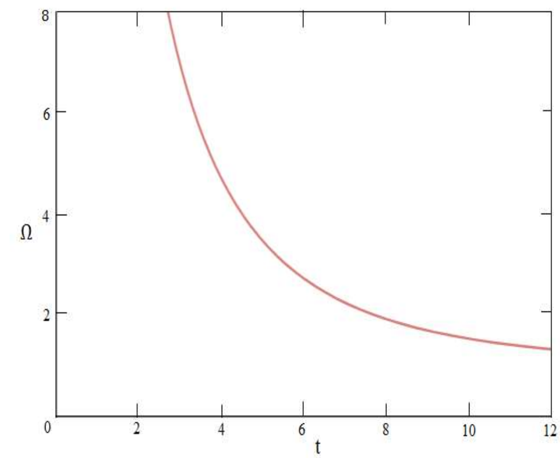


Fig. 5 Plot of total energy density vs. cosmic time t graph with $\alpha=1, D=100, \beta=.05, \gamma=1, \xi=1, t_0=13.8$

Fig. 5 shows the variation of total energy density vs. cosmic time *t.* The total energy density approaches 1. So, our model approaches a flat, isotropic universe at late time.

Now, from Eqns. (21) and (34), the EOS parameter is obtained as

$$\omega_{HDE} = -1 - \frac{-2\alpha\gamma t\left(\frac{\gamma}{t}+\frac{\xi}{t_0}\right)+2\beta\gamma}{3\left(\frac{\gamma}{t}+\frac{\xi}{t_0}\right)\left[\alpha t^3\left(\frac{\gamma}{t}+\frac{\xi}{t_0}\right)^2-\beta\gamma t\right]} \tag{39}$$

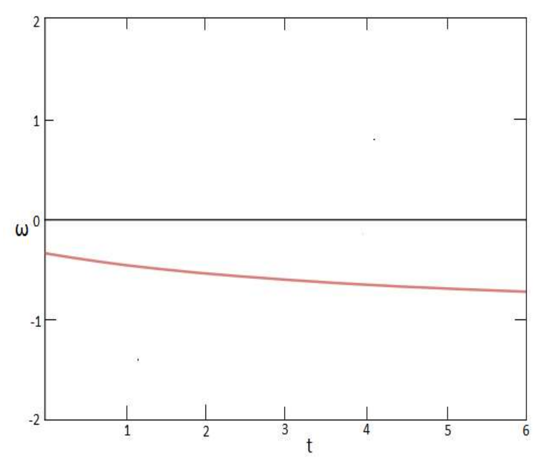


Fig. 6 Plot of EoS parameter vs. cosmic time t graph with $\alpha=1, \beta=.05, \gamma=1, \xi=1, t_0=13.8$

Fig. 6 represents the Eos parameter vs. cosmic time *t* which decreases rapidly and approaches -1 asymptotically. So, our model represents a ΛCDM model for late universe.

### 5. Correspondence between the Holographic and Polytropic gas model of dark energy

The equation of state parameter of polytropic gas is given by

$$P_{pg} = K\rho_{pg}^{1+\frac{1}{\eta}} \tag{40}$$

Where, $K$ and $\eta$ are the polytropic constant and the polytropic index respectively (Christensen-Dalsgaard 2004).
The energy density of polytropic gas is defined as

$$\rho_{pg} = (Ba^{\frac{3}{\eta}} - K)^{-\eta} \tag{41}$$

Where, $B$ is the positive constant of integration and $a$ is the scale factor. It can be seen that the polytropic index $\eta$ should be even to obtain positive energy density.
Using equations (40) and (41), we find the EoS parameter as

$$\omega_{pg} = \frac{P_{pg}}{\rho_{pg}} = -1 - \frac{Ba^{\frac{3}{\eta}}}{K - Ba^{\frac{3}{\eta}}} \tag{42}$$

If polytropic gas is treated as an ordinary scalar field then the energy density and pressure of the scalar field are given by

$$\rho_{\phi} = \frac{\dot{\phi}^2}{2} + V(\phi) \tag{43}$$

$$P_{\phi} = \frac{\dot{\phi}^2}{2} - V(\phi) \tag{44}$$

Where, an over dot denotes the derivative with respect to the cosmic time $t$.
Now, using equations (40), (41), (43) and (44) we obtain the scalar potential and the kinetic energy terms for the polytropic gas model as

$$V(\phi) = \frac{\frac{1}{2}Ba^{\frac{3}{\eta}} - K}{(Ba^{\frac{3}{\eta}} - K)^{\eta+1}} \tag{45}$$

$$\dot{\phi}^2 = \frac{Ba^{\frac{3}{\eta}}}{(Ba^{\frac{3}{\eta}} - K)^{\eta+1}} \tag{46}$$

To establish the correspondence between the holographic dark energy with polytropic gas dark energy model, we compare the holographic dark energy density with the energy density of polytropic gas model and also equate the EoS parameters of both the models. We assume that the holographic dark energy density of our model is equivalent to the energy density of polytropic gas.
Equating equations (33) and (41), we get

$$(Ba^{\frac{3}{\eta}} - K)^{-\eta} = 3[\alpha\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)^2 - \frac{\beta\gamma}{t^2}] \tag{47}$$

Again comparing Eqns. (39) and (42), we get

$$-1 - \frac{Ba^{\frac{3}{\eta}}}{K - Ba^{\frac{3}{\eta}}} = -1 - \frac{2\alpha\gamma t\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right) + 2\beta\gamma}{3\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)[\alpha t^3\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)^2 - \beta\gamma t]} \tag{48}$$

Now, solving Eqns. (47) and (48), we get

$$B = \frac{3^{\frac{-1-\eta}{\eta}}[a_0(\frac{t}{t_0})^{\gamma} e^{\xi(\frac{t}{t_0}-1)}]^{\frac{-3}{\eta}}\left[-2\alpha\gamma t\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right) + 2\beta\gamma\right][\alpha\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)^2 - \frac{\beta\gamma}{t^2}]^{\frac{-1}{\eta}}}{\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)[\alpha t^3\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)^2 - \beta\gamma t]} \tag{49}$$

$$K = \frac{3^{\frac{-1-\eta}{\eta}}[a_0(\frac{t}{t_0})^{\gamma} e^{\xi(\frac{t}{t_0}-1)}]^{\frac{-3}{\eta}}\left[-2\alpha\gamma t\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right) + 2\beta\gamma\right][\alpha\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)^2 - \frac{\beta\gamma}{t^2}]^{\frac{-1}{\eta}}}{\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)[\alpha t^3\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)^2 - \beta\gamma t]} a^{\frac{3}{\eta}} - 3^{\frac{-1}{\eta}}[\alpha\left(\frac{\gamma}{t} + \frac{\xi}{t_0}\right)^2 - \frac{\beta\gamma}{t^2}]^{\frac{-1}{\eta}} \tag{50}$$

Now, using Eqns. (49) and (50) in (45) and (46), we can find the kinetic energy term and the potential of the polytropic gas dark energy model. This type of potential can produce an accelerated expansion of the Universe. Thus a correspondence between the holographic dark energy and polytropic gas can be established. We can also describe holographic dark energy by making use of polytropic gas.

### 6. Conclusion

In this work, we study a spatially homogeneous and anisotropic Bianchi type I universe filled with cold dark matter and HDE. To obtain the exact solution of Einstein's field equations, we consider the hybrid expansion law proposed by Akarsu *et al.* [46], which yields power-law and exponential function. We also discuss some physical and geometrical properties of the model.
We observe that

- The universe starts with a zero volume at $t = 0$.
- The average Hubble parameter $H$ is a decreasing function of time and at $t \to \infty$, $\frac{dH}{dt} \to 0$.
- From Fig. 2, we see that the deceleration parameter (DP) decreases rapidly and approaches $-1$ asymptotically, which shows de-Sitter like expansion at late time.
- $\frac{\sigma^2}{\theta^2} \neq 0$ as well as the anisotropy parameter $A \neq 0$ except at $M = 0$, which implies that our model is anisotropic at all times except when $M = 0$ i.e. the Universe is isotropic only for $M = 0$. But from Fig. 3, we see that anisotropic parameter decreases as time evolves and tends to zero. Hence, we can

conclude that the anisotropy of our universe dies out in the course of evolution to reach the present isotropic phase.

- Fig. 4 shows that both holographic dark energy density and cold dark matter density decrease as time evolves and cold dark matter density tends to zero at late time.
- Fig. 5 shows that for large *t,* the total energy density approaches 1. So, our model approaches a flat, isotropic universe at late time.
- Fig. 6 shows that the EoS parameter of our model is never positive throughout the evolution of the Universe. It decreases rapidly and approaches $-1$ asymptotically. So, our model represents a ΛCDM model for late universe.

Further, we establish a correspondence between holographic dark energy and the polytropic gas model and reconstruct the potential of the polytropic scalar field as well as the dynamics of the scalar field according to the evolution of the holographic dark energy.

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