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Two-fluid Scenario for Bianchi Type-II, VIII & IX Dark Energy Cosmological Models in Brans-Dicke Theory

V. U. M. Rao*,[1], K. V. S. Sireesha[2]
[1]*Department of Applied Mathematics, Andhra University, Visakhapatnam, India*
[2]*Department of Engineering Mathematics, GITAM University, Visakhapatnam, India*

Spatially homogeneous Bianchi type-II, VIII & IX cosmological models filled with barotropic fluid and dark energy are obtained in a scalar tensor theory of gravitation proposed by Brans and Dicke in 1961. We consider the cases when dark energy is minimally coupled to barotropic fluid and when it is in direct interaction with it. In both cases, the equation of state (EoS) parameter ω_{de} changing from $\omega_{de} > -1$ to $\omega_{de} < -1$, which is consistent with the recent observation. Some important features of these models are also discussed.

1. Introduction

In the study of modern cosmology, we consider that the total energy density of the Universe is dominated by the densities of two components: the dark matter and the dark energy. The recent observational data strongly motivate to study general properties of cosmological models containing more than one fluid. These universes are modeled with perfect fluids and with mixtures of non-interacting fluids under the assumption that there is no energy transfer among the components. But, such scenarios are not confirmed by observational data. This motivates us to study cosmological models containing fluids which interact with each other. In recent years there has been immense interest in cosmological models with dark energy in general relativity because of the fact that our observable universes is undergoing a phase of accelerated expansion that has been confirmed by several cosmological observations such as type 1a supernova by several authors [2-8]. Caldwell [9] and Huange [10] have discussed cosmic microwave background (CMB) anisotropy and Daniel et al. [11] have studied large scale structure and strongly indicate that dark energy dominates the present universe, causing cosmic acceleration. Based on these observations, cosmologists have accepted the idea of dark energy, which is a fluid with negative pressure making up around 70% of the energy content of the present universe and to be responsible for this acceleration due to repulsive gravitation. Cosmologists have proposed many candidates for dark energy to fit the current observations such as cosmological constant, tachyon, quintessence, phantom and so on. Evolution of the equation of state (EoS) of dark energy $\omega_D = \frac{p_D}{\rho_D}$ transfers from $\omega_D > -1$ in the near past (quintessence region) to $\omega_D < -1$ at recent stage (phantom region). Akarsu and Kilinc [12,13], Yadav [14], Yadav and Yadav [15] Pradhan et al. [16,17], Pradhan and Amirhashchi [18] and Yadav et al. [19] have investigated different aspects of dark energy models in general relativity with variable EoS parameter. The concept of dark energy was proposed for understanding this currently accelerating expansion of the Universe, and then its existence was confirmed by several high precision observational experiments ([20-22]), especially the Wilkinson Microwave Anisotropy Probe (WMAP) satellite experiment. The WMAP shows that dark energy occupies about 73% of the energy of the Universe, and dark matter about 23%. The usual baryon matter, which can be described by our known particle theory, occupies only about 4% of the total energy of the universe. Measurements as of 2008, with the greatest weight coming from the combination of supernovae with either cosmic microwave background or baryon acoustic oscillation data, show that dark energy makes up 72 $\pm$ 3% of the total energy density of the Universe, and its equation of state averaged over the last 7 billion years is ω = 1.00 ± 0.1. This is consistent with the simplest picture, the cosmological constant, but also with a great many scenarios of time varying dark energy or extended gravity theories. In order to explain why the cosmic acceleration happens, many theories have been

*umrao57@hotmail.com

proposed. Although theories of trying to modify Einstein equations constitute a big part of these attempts, the main stream explanation for this problem, however, is known as theories of dark energies. This motivates us to study cosmological models containing fluids which interact with each other. Tolman [23] and Davidson [24] have considered the interaction between dust-like matter and radiation. Gromov et al. [25] have studied cosmological models with decay of massive particles into radiation or with matter creation. Cataldo et al. [26] have considered the simplest non-trivial cosmological scenarios for an interacting mixture of two cosmic fluids described by power-law scale factors, i.e., the expansion as a power-law in time. An interacting two-fluid scenario for dark energy in an FRW universe has been studied by Amirhashchi et al. [27]. Whereas, an interacting and non-interacting two-fluid scenario for dark energy in an FRW universe with constant deceleration parameter have been described by Pradhan et al. [28]. Adhav et al. [29] have investigated interacting cosmic fluids in LRS BianchiType-I cosmological models. Saha et al. [30] have obtained two-fluid scenario for dark energy models in an FRWuniverse. Adhav et al. [31] have studied Kaluza-Klein interacting cosmic fluid cosmological model. Reddy and Santhi Kumar [32] have discussed two-fluid scenario for dark energy model in a scalar-tensor theory of gravitation. Amirhashchi et al. [33] have studied interacting two-fluid viscous dark energy models in a non-flat universe.

Brans-Dicke [1] theory of gravitation is a natural extension of general relativity, which introduces an additional scalar field ϕ besides the metric tensor g_{ij} and a dimensionless coupling constant ω. The Brans - Dicke field equations for combined scalar and tensor field are given by

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}(\phi_{i;j} - g_{ij}\phi_{;k}{}^{,k}) \tag{1}$$

and

$$\phi_{;k}{}^{,k} = 8\pi(3+2\omega)^{-1}T \tag{2}$$

Where, $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is an Einstein tensor, R is the scalar curvature, ω and n are constants, T_{ij} is the stress energy tensor of the matter and comma and semicolon denote partial and covariant differentiation respectively.

Also, we have energy – conservation equation

$$T^{ij}_{;j} = 0 \tag{3}$$

This equation is a consequence of the field equations (1) and (2).

Several aspects of Brans-Dicke cosmology have been extensively investigated by many authors. The work of Singh and Rai [34] gives a detailed survey of Brans-Dicke cosmological models discussed by several authors. Rao et al. [35] have obtained exact Bianchi type-V perfect fluid Cosmological models in Brans-Dicke theory of gravitation. Rao et al. [36] have obtained axially symmetric string cosmological models in Brans-Dicke theory of gravitation. Rao and Vijaya Santhi [37] have discussed Bianchi type-II, VIII and IX magnetized cosmological models in Brans-Dicke theory of gravitation. Rao and Sireesha [38,39] have studied a higher-dimensional string cosmological model in a scalar-tensor theory of gravitation and Bianchi type-II, VIII and IX string cosmological models with bulk viscosity in Brans-Dicke theory of gravitation. Rao et al. [40] have obtained LRS Bianchi type-I dark energy cosmological model in Brans-Dicke theory of gravitation.

Bianchi type space-times play a vital role in understanding and description of the early stages of evolution of the universe. In particular, the study of Bianchi types II, VIII & IX universes is important because familiar solutions like FRW universe with positive curvature, the de Sitter universe, the Taub-Nut solutions etc. correspond to Bianchi type II, VIII & IX space-times. Rao et al. [41] have studied Bianchi types II, VIII & IX string cosmological models with bulk viscosity in a theory of gravitation. Rao et al. [42] have discussed Perfect fluid cosmological models in a modified theory of gravity.

In this paper, we will discuss spatially homogeneous Bianchi type-II, VIII & IX cosmological models filled with barotropic fluid and dark energy in a scalar-tensor theory of gravitation proposed by Brans and Dicke [1].

2. Metric and Energy Momentum Tensor

We consider a spatially homogeneous Bianchi type-II, VIII & IX metrics of the form

$$ds^2 = dt^2 - R^2\left[d\theta^2 + f^2(\theta)d\phi^2\right] - S^2\left[d\varphi + h(\theta)d\phi\right]^2 \quad (4)$$

Where, (θ, ϕ, φ) are the Eulerian angles, R and S are functions of t only.

It represents:

Bianchi type-II if $f(\theta) = 1$ and $h(\theta) = \theta$
Bianchi type-VIII if $f(\theta) = Cosh\theta$ and $h(\theta) = Sinh\theta$
Bianchi type- IX if $f(\theta) = Sin\theta$ and $h(\theta) = Cos\theta$

The energy momentum tensor for a bulk viscous fluid containing one dimensional string is

$$T_{ij} = (\rho_{tot} + p_{tot})u_i u_j - p_{tot} g_{ij} \quad (5)$$

Where, $\rho_{tot} = \rho_m + \rho_D$ and $p_{tot} = p_m + p_D$. Here ρ_m and p_m are energy density and pressure of the barotropic fluid and ρ_D and p_D are energy density and pressure of dark fluid, respectively, u^i is the four-velocity of the fluid and $g_{ij}u^i u^j = 1$

In a commoving coordinate system, we get

$$T_1^1 = T_2^2 = T_3^3 = -p_{tot}, T_4^4 = \rho_{tot}$$
$$\text{and} \quad T_j^i = 0 \quad \text{for} \quad i \neq j \quad (6)$$

Where, quantities ρ_{tot} and p_{tot} are functions of t only.

3. Solutions of Field equations

Now with the help of Eqns. (5) & (6), the field equations (1) for the metric in Eqn. (4) can be written as

$$\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} + \frac{S^2}{4R^4} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{R}\dot{\phi}}{R\phi} + \frac{\dot{S}\dot{\phi}}{S\phi} = -\frac{8\pi\, p_{tot}}{\phi} \quad (7)$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2 + \delta}{R^2} - \frac{3S^2}{4R^4} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\ddot{\phi}}{\phi} + 2\frac{\dot{R}\dot{\phi}}{R\phi} = -\frac{8\pi\, p_{tot}}{\phi} \quad (8)$$

$$2\frac{\dot{R}\dot{S}}{RS} + \frac{\dot{R}^2 + \delta}{R^2} - \frac{S^2}{4R^4} - \frac{\omega\dot{\phi}^2}{2\phi^2} + 2\frac{\dot{R}\dot{\phi}}{R\phi} + \frac{\dot{S}\dot{\phi}}{S\phi} = \frac{8\pi\rho_{tot}}{\phi} \quad (9$$

$$\left(\frac{\dot{S}}{S} - \frac{\dot{R}}{R}\right)\frac{h(\theta)\dot{\phi}}{\phi} = 0 \quad (10)$$

$$\ddot{\phi} + \dot{\phi}(2\frac{\dot{R}}{R} + \frac{\dot{S}}{S}) = \frac{8\pi}{(3+2\omega)}(\rho_{tot} - 3p_{tot}) \quad (11)$$

Here, the over head dot denotes differentiation with respect to t.

When $\delta = 0, -1$ & $+1$ the field equations (7) to (11) correspond to the Bianchi types II, VIII & IX universes, respectively.

By taking the transformation $R = e^\alpha$, $S = e^\beta$ and $dt = R^2 S\, dT$, the above field equations (7) to (11) can be written as

$$\alpha'' + \beta'' - \alpha'^2 - 2\alpha'\beta' + \frac{e^{4\beta}}{4} + \frac{\omega\phi'^2}{2\phi^2} - \frac{\alpha'\phi'}{\phi} + \frac{\phi''}{\phi} = \frac{-8\pi p_{tot}}{\phi}e^{4\alpha+2\beta} \quad (12)$$

$$2\alpha'' - \alpha'^2 - 2\alpha'\beta' - \frac{3e^{4\beta}}{4} + \delta e^{(2\alpha+2\beta)} + \frac{\omega\phi'^2}{2\phi^2} - \frac{\beta'\phi'}{\phi} + \frac{\phi''}{\phi} = -\frac{8\pi p_{tot}}{\phi}e^{4\alpha+2\beta} \quad (13)$$

$$2\alpha'\beta' + \alpha'^2 - \frac{e^{4\beta}}{4} + \delta e^{(2\alpha+2\beta)} - \frac{\omega\phi'^2}{2\phi^2} + \frac{\beta'\phi'}{\phi} + 2\frac{\alpha'\phi'}{\phi} = \frac{8\pi\rho_{tot}}{\phi}e^{4\alpha+2\beta} \quad (14)$$

$$(\alpha' - \beta')\frac{h(\theta)\phi'}{\phi} = 0 \quad (15)$$

$$\phi'' = \frac{8\pi}{3+2\omega}(\rho - 3p_{tot})e^{4\alpha+2\beta} \quad (16)$$

Here, the over head dash denotes differentiation with respect to 'T' and α, β are functions of 'T' only.

Since we are considering the Bianchi type-II, VIII & IX metrics, we have $h(\theta) = \theta, h(\theta) = \sinh\theta \,\&\, h(\theta) = \cos\theta$, respectively.

Therefore, from Eqn. (15) we will get the following possible cases with $h(\theta) \neq 0$

$$\begin{aligned}&(1)\ \alpha' - \beta' = 0 \ \text{ and } \ \phi' \neq 0\\&(2)\ \alpha' - \beta' \neq 0 \ \text{ and } \ \phi' = 0\\&(3)\ \alpha' - \beta' = 0 \ \text{ and } \ \phi' = 0\end{aligned}$$

From the above three possibilities, we will consider only the first possibility, since in other two cases we will get cosmological models in general relativity.

3.1. Cosmological models in Brans-Dicke theory

We will get cosmological models in Brans-Dicke theory only in case of $\alpha' - \beta' = 0$ and $\phi' \neq 0$.

If $\alpha' - \beta' = 0$, then we get $\alpha = \beta + c$.

Without loss of generality by taking the constant of integration, $c = 0$, we get

$$\alpha = \beta \tag{17}$$

Using Eqn. (17), the above field equations (12) to (16) can be written as

$$2\beta'' - 3\beta'^2 + \frac{e^{4\beta}}{4} + \frac{\omega\phi'^2}{2\phi^2} - \frac{\beta'\phi'}{\phi} + \frac{\phi''}{\phi} = \frac{-8\pi p_{tot}}{\phi} e^{6\beta} \tag{18}$$

$$2\beta'' - 3\beta'^2 - \frac{3e^{4\beta}}{4} + \delta e^{4\beta} + \frac{\omega\phi'^2}{2\phi^2} - \frac{\beta'\phi'}{\phi} + \frac{\phi''}{\phi} = -\frac{8\pi\, p_{tot}}{\phi} e^{6\beta} \tag{19}$$

$$3\beta'^2 - \frac{e^{4\beta}}{4} + \delta e^{4\beta} - \frac{\omega\phi'^2}{2\phi^2} + \frac{3\beta'\phi'}{\phi} = \frac{8\pi\rho_{tot}}{\phi} e^{6\beta} \tag{20}$$

$$\phi'' = \frac{8\pi}{3+2\omega}(\rho_{tot} - 3p_{tot})e^{6\beta} \tag{21}$$

The field equations (18) to (21) are four independent equations with five unknowns.

From Eqns. (18) - (21), we have

$$3\beta'' - 3\beta'^2 - \frac{e^{4\beta}}{4} + \delta e^{4\beta} + \frac{\omega\phi'^2}{2\phi^2} - \frac{\omega\phi''}{\phi} = 0 \tag{22}$$

From Eqn. (22), we get

$$e^{\beta} = (aT + b)^{-1/2} \tag{23}$$

$$\phi = (aT + b)^{m} \tag{24}$$

Where, $m = \dfrac{2\omega a \pm \sqrt{4\omega^2 a^2 + 2\omega(3a^2 + 4\delta - 1)}}{2\omega a}$

The Bianchi Identity $G^{ij}_{;j} = 0$ leads to $T^{ij}_{;j} = 0$ yields

$$\dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) = 0 \tag{25}$$

The EoS of the barotropic fluid and dark field are respectively given by

$$w_m = \frac{p_m}{\rho_m} \ \text{ and } \ w_D = \frac{p_D}{\rho_D} \tag{26}$$

In the following sections we deal with two cases: (i) non-interacting two-fluid model and (ii) interacting two-fluid model.

3.2. Non-interacting two-fluid model

First, we consider that two fluids do not interact with each other. Therefore, the general form of conservation Eqn. (25) leads us to write the conservation equation for the dark and barotropic fluid separately as

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0 \tag{27}$$

and

$$\dot{\rho}_D + 3H(\rho_D + p_D) = 0 \tag{28}$$

Here there is, of course, a structural difference between Eqns. (27) and (28). Because Eqn. (27) is in the form of ω_m, which is constant, and hence it is integrable. But Eqn. (28) is a function of ω_D. Accordingly, ρ_D and p_D are also function of ω_D. Therefore, we cannot integrate Eqn. (28) as it is a function of ω_D, which is an unknown time dependent parameter.

Integration of Eqn. (27) leads to

$$8\pi\rho_m = 8\pi\rho_0 (aT + b)^{\frac{-3(1+w_m)}{2}} \tag{29}$$

3.3. Bianchi type-II ($\delta = 0$) cosmological model

From Eqns. (20), (23), (24) and (29) we get

$$8\pi\rho_D = \left[\frac{(3a^2-1)-2ma^2(\omega m+3)}{4}\right](aT+b)^{m+1} - 8\pi\rho_0(aT+b)^{\frac{-3(1+w_m)}{2}} \tag{30}$$

From Eqns. (26) and (29) we get

$$8\pi\, p_m = 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m)}{2}} \tag{31}$$

From Eqns. (19), (23), (24) and (31) we get

$$8\pi\, p_D = \frac{[2ma^2[1-(\omega+2)m]-(a^2-3)]}{4}(aT+b)^{m+1} - 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m)}{2}} \tag{32}$$

From Eqns. (26), (30) and (32) we get

$$w_D = \frac{\frac{[2ma^2[1-(\omega+2)m]-(a^2-3)]}{4}(aT+b)^{m+1} - 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m)}{2}}}{\left[\frac{(3a^2-1)-2ma^2(\omega m+3)}{4}\right](aT+b)^{m+1} - 8\pi\rho_0 (aT+b)^{\frac{-3(1+w_m)}{2}}} \tag{33}$$

3.4. Interacting two-fluid model

Secondly, we consider the interaction between dark energy and barotropic fluids. For this purpose, we can write the continuity equations for dark fluid and barotropic fluids as

$$\dot{\rho}_m + 3H(\rho_m + p_m) = Q \tag{34}$$

and

$$\dot{\rho}_D + 3H(\rho_D + p_D) = -Q \tag{35}$$

The quantity Q expresses the interaction between dark energy components. Since we are interested in an energy transfer from the dark energy to dark matter, we consider $Q>0$, which ensures that the second law of thermodynamics is fulfilled [43]. Here we emphasize that continuity Eqns. (34) and (35) imply that interaction term Q should be proportional to a quantity with units of inverse time i.e., $Q \propto \frac{1}{t}$. Therefore, a first and natural candidate can be the Hubble factor H multiplied by the energy density. Following Amendola et al. [44] and Guo et al. [45], we consider

$$Q = 3H\sigma\rho_m \tag{36}$$

Where, σ is a coupling constant.

Using Eqn. (36) in Eqn. (34) and after integrating, we obtain

$$8\pi\rho_m = 8\pi\rho_0 (aT+b)^{\frac{-3(1+w_m-\sigma)}{2}} \tag{37}$$

From Eqns. (20), (23), (24) and (37) we get

$$8\pi\rho_D = \left[\frac{(3a^2-1)-2ma^2(\omega m+3)}{4}\right](aT+b)^{m+1} - 8\pi\rho_0(aT+b)^{\frac{-3(1+w_m-\sigma)}{2}} \tag{38}$$

From Eqns. (26) and (37) we get

$$8\pi\, p_m = 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m-\sigma)}{2}} \tag{39}$$

From Eqns. (19), (23), (24) and (39) we get

$$8\pi\, p_D = \frac{[2ma^2[1-(\omega+2)m]-(a^2-3)]}{4}(aT+b)^{m+1} - 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m-\sigma)}{2}} \tag{40}$$

From Eqns. (26), (38) and (40) we get

$$w_D = \frac{\frac{[2ma^2(1-(\omega+2)m)-(a^2-3)]}{4}(aT+b)^{m+1} - 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m-\sigma)}{2}}}{\left[\frac{(3a^2-1)-2ma^2(\omega m+3)}{4}\right](aT+b)^{m+1} - 8\pi\rho_0 (aT+b)^{\frac{-3(1+w_m-\sigma)}{2}}} \tag{41}$$

The metric in Eqn. (4), in this case, can be written as

$$\begin{aligned} ds^2 &= (aT+b)^{\frac{-3}{2}} dT^2 - (aT+b)^{-1}\left(d\theta^2 + d\phi^2\right) \\ &\quad - (aT+b)^{-1}\left(d\varphi + \theta\, d\phi\right)^2 \end{aligned} \tag{42}$$

Thus Eqn. (42) together with Eqns. (30) to (33) and (38) to (41) constitutes a Bianchi type-II two fluid cosmological model in Brans-Dicke [1] scalar tensor theory of gravitation.

3.5. Bianchi type-VIII ($\delta = -1$) cosmological model

From Eqns. (20), (23), (24) and (29) we get

$$\begin{aligned} 8\pi\rho_D &= \frac{[(3a^2-5)-2ma^2(\omega m+3)]}{4}(aT+b)^{m+1} \\ &\quad - 8\pi\rho_0 (aT+b)^{\frac{-3(1+w_m)}{2}} \end{aligned} \tag{43}$$

From Eqns. (26) and (29) we get

$$8\pi\, p_m = 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m)}{2}} \tag{44}$$

From Eqns. (19), (23), (24) and (44) we get

$$\begin{aligned} 8\pi\, p_D &= \frac{[2m(1-(\omega+2)m)a^2-(a^2-7)]}{4}(aT+b)^{m+1} \\ &\quad - 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m)}{2}} \end{aligned} \tag{45}$$

From Eqns. (26), (30) and (32) we get

$$w_D = \frac{\frac{[2ma^2(1-(\omega+2)m)-(a^2-7)]}{4}(aT+b)^{m+1} - 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m)}{2}}}{\left[\frac{(3a^2-5)-2ma^2(\omega m+3)}{4}\right](aT+b)^{m+1} - 8\pi\rho_0 (aT+b)^{\frac{-3(1+w_m)}{2}}} \tag{46}$$

3.6. Interacting two-fluid model

We consider now the interaction between dark energy and barotropic fluids. For this purpose, we can write the continuity equations for dark fluid and barotropic fluids as

$$\dot{\rho}_m + 3H(\rho_m + p_m) = Q \tag{47}$$

and

$$\dot{\rho}_D + 3H(\rho_D + p_D) = -Q \tag{48}$$

The quantity Q expresses the interaction between the dark energy components. Since we are interested in an energy transfer from the dark energy to dark matter, we consider $Q>0$, which ensures that the second law of thermodynamics is fulfilled [43]. Here we emphasize that the continuity Eqns. (47) and (48) imply that interaction term Q should be proportional to a quantity with units of inverse of time i.e., $Q \propto \frac{1}{t}$. Therefore, a first and natural candidate can be the Hubble factor H multiplied with the energy density. Following Amendola et al. [44] and Guo et al. [45], we consider

$$Q = 3H\sigma\rho_m \tag{49}$$

Where, σ is a coupling constant.

Using Eqn. (49) in Eqn. (48) and after integrating, we obtain

$$8\pi\rho_m = 8\pi\rho_0\,(aT+b)^{\frac{-3(1+w_m-\sigma)}{2}} \tag{50}$$

From Eqns. (20), (23), (24) and (50) we get

$$8\pi\rho_D = \frac{[(3a^2-5)-2ma^2(\omega m+3)]}{4}(aT+b)^{m+1} - 8\pi\rho_0(aT+b)^{\frac{-3(1+w_m-\sigma)}{2}} \tag{51}$$

From Eqns. (26) and (50) we get

$$8\pi\, p_m = 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m-\sigma)}{2}} \tag{52}$$

From Eqns. (19), (23), (24) and (52) we get

$$8\pi\, p_D = \frac{[2ma^2(1-(\omega+2)m)-(a^2-7)]}{4}(aT+b)^{m+1} - 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m-\sigma)}{2}} \tag{53}$$

From Eqns. (26), (51) and (53) we get

$$w_D = \frac{\frac{[2ma^2(1-(\omega+2)m)-(a^2-7)]}{4}(aT+b)^{m+1} - 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m-\sigma)}{2}}}{\left[\frac{(3a^2-5)-2ma^2(\omega m+3)}{4}\right](aT+b)^{m+1} - 8\pi\rho_0(aT+b)^{\frac{-3(1+w_m-\sigma)}{2}}} \tag{54}$$

The metric in eqn. (4), in this case, can be written as

$$ds^2 = (aT+b)^{\frac{-3}{2}}dT^2 - (aT+b)^{-1}\left(d\theta^2 + \cosh^2\theta\, d\phi^2\right) - (aT+b)^{-1}\left(d\varphi + \sinh\theta\, d\phi\right)^2 \tag{55}$$

Thus, Eqn. (55) together with Eqns. (43) to (46) and Eqns. (50) to (54) constitutes a Bianchi type-VIII two fluid cosmological model in Brans-Dicke [1] scalar tensor theory of gravitation.

3.7. Bianchi type-IX ($\delta = -1$) cosmological model

From Eqns. (20), (23), (24) and (29) we get

$$8\pi\rho_D = \frac{[3(a^2+1)-2ma^2(\omega m+3)]}{4}(aT+b)^{m+1} - 8\pi\rho_0\,(aT+b)^{\frac{-3(1+w_m)}{2}} \tag{56}$$

From Eqns. (26) and (29) we get

$$8\pi\, p_m = 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m)}{2}} \tag{57}$$

From Eqns. (19), (23), (24) and (57) we get

$$8\pi\, p_D = \frac{[2ma^2\{1-(\omega+2)m\}-(a^2+1)]}{4}(aT+b)^{m+1} - 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m)}{2}} \tag{58}$$

From Eqns. (26), (56) and (58) we get

$$w_D = \frac{\frac{[2ma^2\{1-(\omega+2)m\}-(a^2+1)]}{4}(aT+b)^{m+1} - 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m)}{2}}}{\left[\frac{3(a^2+1)-2ma^2(\omega m+3)}{4}\right](aT+b)^{m+1} - 8\pi\rho_0 (aT+b)^{\frac{-3(1+w_m)}{2}}} \tag{59}$$

3.8. Interacting two-fluid model

Secondly, we consider the interaction between dark energy and barotropic fluids. For this purpose, we can write the continuity equations for dark fluid and barotropic fluids as

$$\dot{\rho}_m + 3H(\rho_m + p_m) = Q \tag{60}$$

and

$$\dot{\rho}_D + 3H(\rho_D + p_D) = -Q \tag{61}$$

The quantity Q expresses the interaction between the dark energy components. Since we are interested in an energy transfer from the dark energy to dark matter, we consider $Q>0$ which ensures that the second law of thermodynamics is fulfilled [43]. Here we emphasize that the continuity Eqns. (60) and (61) imply that the interaction term (Q) should be proportional to a quantity with units of inverse of time i.e., $Q \propto \frac{1}{t}$. Therefore, a first and natural candidate can be the Hubble factor H multiplied with the energy density. Following Amendola et al. [44] and Guo et al. [45], we consider

$$Q = 3H\sigma\rho_m \tag{62}$$

Where, σ is a coupling constant.

Using Eqn. (62) in Eqn. (60) and after integrating, we obtain

$$8\pi\rho_m = 8\pi\rho_0 (aT+b)^{\frac{-3(1+w_m-\sigma)}{2}} \tag{63}$$

From Eqns. (20), (23), (24) and (63) we get

$$8\pi\rho_D = \frac{[3(a^2+1)-2ma^2(\omega m+3)]}{4}(aT+b)^{m+1} - 8\pi\rho_0 (aT+b)^{\frac{-3(1+w_m-\sigma)}{2}} \tag{64}$$

From Eqns. (26) & (63) we get

$$8\pi p_m = 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m-\sigma)}{2}} \tag{65}$$

From Eqns. (19), (23), (24) and (65) we get

$$8\pi p_D = \frac{[2ma^2[1-(\omega+2)m]-(a^2+1)]}{4}(aT+b)^{m+1} - 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m-\sigma)}{2}} \tag{66}$$

From Eqns. (26), (64) and (66) we get

$$w_D = \frac{\frac{[2ma^2[1-(\omega+2)m]-(a^2+1)]}{4}(aT+b)^{m+1} - 8\pi\rho_0 w_m (aT+b)^{\frac{-3(1+w_m-\sigma)}{2}}}{\frac{[3(a^2+1)-2ma^2(\omega m+3)]}{4}(aT+b)^{m+1} - 8\pi\rho_0 (aT+b)^{\frac{-3(1+w_m-\sigma)}{2}}} \tag{67}$$

The metric (4), in this case can be written as

$$ds^2 = (aT+b)^{\frac{-3}{2}} dT^2 - (aT+b)^{-1}\left(d\theta^2 + \cosh^2\theta\, d\phi^2\right) - (aT+b)^{-1}\left(d\varphi + \sinh\theta\, d\phi\right)^2 \tag{68}$$

Thus Eqn. (67) together with (56) to (59) and (63) to (67) constitutes a Bianchi type-IX two fluid cosmological model in Brans-Dicke [1] scalar tensor theory of gravitation.

4. Some Other Important Properties of the Models

The spatial volume for the models is

$$V=(-g)^{\frac{1}{2}}=(aT+b)^{\frac{-3}{2}}f(\theta) \tag{69}$$

The average scale factor for the model is

$$a(t)=V^{\frac{1}{3}}=(aT+b)^{\frac{-1}{2}}[f(\theta)]^{1/3} \tag{70}$$

The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta=u^i{}_{,i}=\frac{-3}{2}\frac{a}{(aT+b)} \tag{71}$$

and the shear scalar σ is given by

$$\sigma^2=\frac{1}{2}\sigma^{ij}\sigma_{ij}=\frac{9}{8}\frac{a^2}{(aT+b)^2} \tag{72}$$

The deceleration parameter q is given by

$$q=(-3\theta^{-2})(\theta_{,i}u^i+\frac{1}{3}\theta^2)=-3 \tag{73}$$

The deceleration parameter appears with negative sign implies accelerating expansion of the universe, which is consistent with the present day observations.

The Hubble's parameter H is given by

$$H=\frac{-1}{2}\frac{a}{(aT+b)} \tag{74}$$

The mean anisotropy parameter A_m is given by

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2=0$$

where

$$\Delta H_i=H_i-H \quad (i=1,2,3) \tag{75}$$

Look-back time-red shift: The look-back time, $\Delta t=t_0-t(z)$ is the difference between the age of the universe at present time (z=0) and the age of the universe when a particular light ray at red shift z, the expansion scalar of the universe $a(t_z)$ is related to a_0 by $1+z=\frac{a_0}{a}$, where a_0 is the present scale factor. Therefore, from (70), we get

$$1+z=\frac{a_0}{a}=\left(\frac{aT_0+b}{aT+b}\right)^{\frac{-1}{2}} \tag{76}$$

This equation can also be expressed as

$$H_0\Delta T=1-(1+z)^2 \tag{77}$$

Where, H_0 is the Hubble's constant.

Luminosity distance:

Luminosity distance is defined as the distance which will preserve the validity of the inverse law for the fall of intensity and, is given by

$$d_L=r_1(1+z)a_0 \tag{78}$$

Where r_1 is the radial coordinate distance of the object at light emission and, is given by

$$r_1=\int_T^{T_0}\frac{1}{a}dT=\frac{2}{3a}(aT_0+b)^{\frac{3}{2}}\left[1-(1+z)^{-3/2}\right] \tag{79}$$

From Eqns. (78) and (79), we get

The luminosity distance

$$d_L=\frac{2}{3a}a_0(1+z)(aT_0+b)^{\frac{3}{2}}\left[1-(1+z)^{-3/2}\right] \tag{80}$$

From Eqns. (79) and (80), we get

The distance modulus

$$D(z)=5\log\left[\frac{2}{3a}a_0(1+z)(aT_0+b)^{\frac{3}{2}}\left[1-(1+z)^{-3/2}\right]\right]+25 \tag{81}$$

The tensor of rotation $w_{ij} = u_{i,j} - u_{j,i}$ is identically zero and hence this universe is non-rotational.

5. Discussion and Conclusions

In this paper we have presented spatially homogeneous Bianchi type - II, VIII & IX two fluid cosmological model in Brans-Dicke (1961) scalar tensor theory of gravitation.

The following are the observations and conclusions.

- The models are always isotropic and have singularity at $T = \frac{-b}{a}$.
- The volume decreases with the increase of time i.e., as $T \to \infty$, the spatial volume vanishes.
- At $T = \frac{-b}{a}$, the expansion scalar θ, shear scalar σ and the Hubble parameter H decreases with the increase of time.
- From (75), one can observe that $A_m = 0$ and this indicates that these universes always expand isotropically.
- The deceleration parameter appears with negative sign implies accelerating expansion of the universe, which is consistent with the present day observations.
- We have obtained expressions for look-back time ΔT, distance modulus $D(z)$ and luminosity distance d_L versus red shift and discussed their significance.
- All the models presented here are isotropic, non-rotating, shearing and also accelerating. Hence they represent not only the early stage of evolution but also the present universe.

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